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Seed perturbations for primordial magnetic fields from MSSM flat directions

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We demonstrate that the MSSM flat directions can naturally account for the seed magnetic fields in the early Universe. The non-zero vacuum expectation value of an MSSM flat direction condensate provides masses to the gauge fields and thereby breaks conformal invariance. During inflation the condensate receives spatial perturbations and $SU(2)\times U(1)_Y$ gauge currents are generated together with (hyper)magnetic fields. When these long wavelength vector perturbations reenter our horizon they give rise to $U(1)_{\rm em}$ magnetic fields with an amplitude of 10^{-30} Gauss, as required by the dynamo mechanism.

A large scale magnetic field with a coherence scale of few kpc and an amplitude of order μG is observed today in galaxies and galactic clusters [1]. The origin of the cosmic magnetic field may well be found in the small primordial magnetic fluctuations in the early Universe, which are amplified by the galactic dynamo mechanism [1, 2]. While various astrophysical processes, such as battery [3] or vorticity effects [4] face an uphill task in sustaining the large scale coherence length [5], there is no shortage of microphysical processes in the early Universe that could generate a magnetic field [6]. The problem is usually the shortness of the magnetic coherence length, which however could be enhanced by the inverse cascade of magnetohydrodynamic turbulence [7].

It has been noted already many years ago that the coherence length problem might be solved by inflation provided conformal invariance gets broken during inflation. Interesting possibilities have been explored where the conformal invariance is broken either via triangle-anomaly of the gauge field stress energy tensor [8], or via the gravitational or anomalous coupling of the photon to the axion [9]. There are also proposals for exciting the massive Z-boson of the Standard Model during inflation and preheating, which could produce $U(1)_{\rm em}$ magnetic field after the electroweak phase transition via hypercharge mixing [10].

In this paper we will discuss a simple mechanism for generating a seed magnetic field during inflation in the context of the Minimally Supersymmetric Standard Model (MSSM). As is well known, MSSM has flat directions, made up of gauge invariant combinations of squarks and sleptons, which may acquire non-vanishing vacuum expectation values (vevs) during inflation, thereby forming homogeneous zero-mode condensates. The condensates may play a significant role in many cosmological phenomena [11], such as generating baryons and dark matter particles by first fragmenting into Q-balls which then decay [12]; it has also been suggested that the origin of all matter and density perturbations could, in principle, be due to MSSM flat directions [13].

By definition, flat directions are the minimum energy configurations even when supersymmetry is broken. Because of the condensate the SM gauge fields obtain non-vanishing mass terms, thereby breaking the conformal invariance. This is not by itself sufficient for generation of magnetic fields, since for the homogenous condensate the gauge field configurations are pure gauge [14]. However, during inflation the flat directions obtain quantum fluctuations from the inflaton induced false vacuum, which perturbs the classical condensate and also induces fluctuations in the gauge degrees of freedom. These may no longer be gauged away. The situation may be compared with plasma physics, where an electromagnetic field is generated in an electrically neutral plasma by virtue of the motion of the electrons. Here the condensate perturbations represent also perturbations in the charge density and by virtue of their motions, generate gauge currents.

While inflation lasts, the gauge field perturbations are stretched outside the horizon. When they eventually reenter they provide us with a seed (hyper)magnetic field, which after electroweak phase transition is projected onto the regular $U(1)_{\rm em}$ magnetic field.

Let us consider one particular flat direction with components carrying $SU(2)\times U(1)_Y$ charges such as $LL\bar{e}$. The field content of this direction is given by

$$L_i = \frac{\phi}{\sqrt{3}}\begin{pmatrix}1\\0\end{pmatrix},\quad L_j = \frac{\phi}{\sqrt{3}}\begin{pmatrix}0\\1\end{pmatrix},\quad \bar{e}_k = \frac{\phi}{\sqrt{3}} \tag{1}$$

where $i\neq j$, ϕ is a complex scalar field and square roots are to obtain canonical normalization. The background gauge fields are assumed to vanish. With the field configuration Eq. (1) it is possible to show that the gauge invariant matter currents

$$\begin{aligned} J_\mu^A &= ig\sum_i\left[\phi_i^\dagger T^A D_\mu\phi_i - (D_\mu\phi_i)^\dagger T^A\phi_i\right],\\ J_\mu^Y &= ig'\sum_i Y_i\left[\phi_i^\dagger D_\mu\phi_i - (D_\mu\phi_i)^\dagger\phi_i\right], \end{aligned} \tag{2}$$

vanish for vanishing gauge fields in a Robertson-Walker background $g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}$, where $a(\eta)$ is the scale factor in comoving coordinates and η the conformal time. The covariant derivative is given by $D_\mu = \partial_\mu - igA^A_\mu T^A - ig'YA_\mu$, where $A_\mu(A^A_\mu)$ is the Abelian (non-Abelian) gauge field corresponding to $U(1)_Y$ ($SU(2)$). The summation in Eq. (2) is over the scalar fields belonging to the representation of the corresponding gauge group.

The corresponding equations of motion are given by

$$\begin{aligned} \eta^{\rho\nu} D^{(Adj)}_\rho F^A_{\mu\nu} &= a(\eta)^2 J^A_\mu \,, \\ \eta^{\rho\nu}\partial_\rho F_{\mu\nu} &= a(\eta)^2 J^Y_\mu \,, \\ \eta^{\mu\nu} D_\mu D_\nu \phi_i + 2\mathcal{H} D_0\phi_i + \frac{\partial V}{\partial \phi_i^\dagger} &= 0 \,. \end{aligned} \tag{3}$$

Here $\mathcal{H}$ is the comoving Hubble expansion rate, and the adjoint covariant derivative is determined by $D^{(Adj)}_\rho F^A_{\mu\nu} = \partial_\rho F^A_{\mu\nu} + gf^{ABC}A^B_\rho F^C_{\mu\nu}$, where f^{ABC} is the structure constant of the $SU(2)$ gauge group, and $F^A_{\mu\nu}$ is the gauge field strength.

Perturbing Eq. (3) to first order we get[1]

$$\begin{aligned} \eta^{\rho\nu}\partial_\rho\left(\partial_\mu \delta A^A_\nu - \partial_\nu \delta A^A_\mu\right) &= a^2 \delta J^A_\mu \\ \eta^{\rho\nu}\partial_\rho\left(\partial_\mu \delta A_\nu - \partial_\nu \delta A_\mu\right) &= a^2 \delta J^Y_\mu \end{aligned} \tag{4}$$

where the background equation $A^A_\mu = A_\mu = 0$ has been used. In calculating the perturbations of the matter currents the perturbations of the flat direction, $\delta\phi$, do not appear since the current vanishes for all values of ϕ for vanishing gauge fields, so only the terms proportional to gauge field perturbations contribute:

$$\begin{aligned} \delta J^A_\mu &= ig\Big[L_i^\dagger T^A(-igT^B\delta A^B_\mu - ig'Y_L\delta A_\mu)L_i \\ &\quad - L_i^\dagger(igT^B\delta A^B_\mu + ig'Y_L\delta A_\mu)T^A L_i + (L_i \to L_j)\Big] \\ \delta J^Y_\mu &= ig'\Big[2Y_L L_i^\dagger(-igT^B\delta A^B_\mu - ig'Y_L\delta A_\mu)L_i \\ &\quad + (L_i \to L_j) - 2ig'Y^2_{\bar{e}}\delta A_\mu|\bar{e}_k|^2\Big] \end{aligned} \tag{5}$$

Inserting the background field configuration of the flat direction Eq. (1) into Eq. (5), we finally obtain

$$\begin{aligned} \delta J^A_\mu &= \frac{1}{3}g^2|\phi|^2\delta A^A_\mu \\ \delta J^Y_\mu &= g'^2|\phi|^2\delta A_\mu \end{aligned} \tag{6}$$

Hence Eq. (6) produces effective mass terms for Eq. (4). The equations of motion are similar to massive scalar field equations of motion and thus in the Coulomb gauge $\delta A^A_0 = \delta A_0 = \partial^i\delta A^A_i = \partial^i\delta A_i = 0$, Eq. (4) may be written as

$$\begin{aligned} (\partial^2_\eta - \partial^j\partial_j)\delta A^A_i + \frac{1}{3}g^2|\phi|^2 a(\eta)^2\delta A^A_i &= 0 \\ (\partial^2_\eta - \partial^j\partial_j)\delta A_i + g'^2|\phi|^2 a(\eta)^2\delta A_i &= 0 \,, \end{aligned} \tag{7}$$

where $i = 1, 2, 3$. Eq. (7) reminds us of a scalar field equation with mass $m = |g\phi|/\sqrt{3}$ and $m' = |g'\phi|$. The solutions can be written as plane wave expansions

$$\delta A^A_i = \sum_{\mathbf{k}}\left(w_k(\eta)e^{A\lambda}_i a_{\mathbf{k}\lambda} + w_k(\eta)^* e^{A\lambda *}_i a^\dagger_{-\mathbf{k}\lambda}\right)e^{i\mathbf{k}\cdot\mathbf{x}} \,, \tag{8}$$

where a corresponding expansion holds for δA_i, too. Here $e^{A\lambda}_i$ are the polarization vectors which obey $k^i e_i = 0$ in the Coulomb gauge. The mode functions, $w_k(\eta)$, can be given in terms of Hankel functions as

$$w_k(\eta) \;=\; \frac{\sqrt{-\pi\eta}}{2(2\pi)^{3/2}} e^{i\left(\nu+\frac{1}{2}\right)\frac{\pi}{2}} H^{(1)}_\nu(-k\eta) \tag{9}$$

and similar expression holds for the mode function of δA_i with $\nu \to \nu'$. In the above expression k is the wavenumber and

$$\nu^2 = \frac{1}{4} - \frac{g^2|\phi|^2}{3H_I^2} \,, \quad \nu'^2 = \frac{1}{4} - \frac{g'^2|\phi|^2}{H_I^2} \,, \tag{10}$$

where H_I is the Hubble expansion rate during inflation. The mode functions (9) reduce to the vacuum mode functions in the limit $\eta \to -\infty$,

$$w^{(vac)}_k(\eta) = \frac{1}{(2\pi)^{3/2}\sqrt{2k}}\, e^{-ik\eta} \tag{11}$$

We are mainly interested in the super Hubble horizon fluctuations of $\delta A_i, \delta A^A_i$. When $k|\eta| \ll 1$, we obtain,

$$\left|\delta A^A_i\right|^2 \;\approx\; \frac{2|\Gamma(\nu)|^2}{(2\pi)^4 k}\left(\frac{k}{2H_I}\right)^{1-2\nu} \,. \tag{12}$$

and a similar expression for $|\delta A_i|^2$ but with $\nu \to \nu'$. The amplitudes of the long wavelength fluctuations are frozen and start to re-enter once inflation comes to an end. For the sake of simplicity we assume here that post-inflationary era does not trigger super-Hubble fluctuations in the gauge fields which would alter our result in Eq. (12). However, note that the vacuum structure is different during inflation and the radiation era[2]. Therefore at late times it is important to match the above mode functions (valid during inflation) to a linear combination

[1] We have neglected the perturbations of metric and non-flat degrees of freedom. In the gauge $\delta A^A_0 = 0$ and $\partial^i\delta A^A_i = 0$ the result for the gauge field equation turns out to be the same.

[2] Note that the flat direction condensate amplitude slides both during inflation and post-inflationary era. Therefore the masses m, m' vary with time. The MSSM flat direction decays late but well before the electroweak scale [15], and by decaying they restore the conformal invariance.

of traveling waves in the radiation dominated epoch via Bugoliubov coefficients, $\alpha_k,\ \beta_k$,

$$\begin{aligned} w_k(\eta) &= \alpha_k w_k^{(vac)}(\eta) + \beta_{-k}^* w_k^{(vac)}(\eta)^* , \\ \partial_\eta w_k(\eta) &= \alpha_k \partial_\eta w_k^{(vac)}(\eta) + \beta_{-k}^* \partial_\eta w_k^{(vac)}(\eta) , \end{aligned} \quad (13)$$

For simplicity let us assume that the transition from inflation to radiation happens instantly, and we also assume that MSSM flat direction vev becomes zero right after the end of inflation so that the condensate disappears instantaneously. Although the MSSM flat direction vev always goes to zero, it need not vanish right away after the end of inflation. However, it is possible to find an example within D-term inflationary models where the flat direction vev can become zero right after the end of inflation; this happens due to a large Hubble induced mass correction which causes the condensate field to roll down to the origin within one Hubble time [16].

In this letter we proceed with the assumption of a sudden transition from inflation to radiation on super horizon scales, which leads to

$$|\beta_k|^2 \approx \frac{|\Gamma(\nu')|^2}{16\pi}\left(\frac{1}{2}-\nu'\right)^2\left(\frac{2H_I}{k}\right)^{1+2\nu'} , \quad (14)$$

and a similar expression for $|\beta_k^A|^2$ with $\nu' \to \nu$. Therefore during the radiation era the amplitude of the gauge fields is enhanced by a factor β_k compared to the Minkowski vacuum. Note that during radiation era, $w_k^{(vac)}(\eta) \propto \sqrt{1/2k}e^{-ik\eta}$. Therefore the final the gauge field spectrum $\propto k^{-1-\nu}$. For $\nu \approx 1/2$, the spectrum is indeed flat, similar to the case of massless scalar field. Similar conclusions were also reached in Ref. [10].

During radiation era, at scales where diffusion can be neglected, the hypercharge field is frozen in the sense that the lines of force move together with the fluid, $A_i,\ A_i^A \propto a(t)^{-2}$.

In our case, the hypercharge field is a linear combination of the fields, $Y_\mu \sim \mathcal{B}_\mu + Z_\mu$. We are mainly concerned with the $U(1)_{\rm em}$ photon field, $\mathcal{B}_\mu$, below the electroweak scale, given by $\mathcal{B}_\mu = \sin\theta_W A_\mu^3 + \cos\theta_W A_\mu$, where $\sin\theta_W \approx 0.23$ is the Weinberg's angle. Since the non-Abelian field A_μ^A obtains a screening mass in a radiation dominated Universe [17], its contribution vanishes and the unscreened photon field is $\mathcal{B}_i \approx \cos\theta_W Y_i$ and the real magnetic field is $B_i = \epsilon_{ijk}\partial_j \mathcal{B}_k$.

Let us now estimate the amplitude of the magnetic field at the length scale l where $k = 2\pi/l \ll H_I$. We obtain

$$B_l \approx \cos\theta_W\ C(\nu')\left(\frac{1}{2}-\nu'\right) H_I^2 (l\ H_I)^{\nu'-3/2} , \quad (15)$$

where

$$C(\nu') = 3\cdot 2^{2\nu'}\Gamma(\nu')\sqrt{\frac{2\Gamma(1-\nu')}{\sqrt{\pi}\Gamma(3/2+\nu')}} . \quad (16)$$

Within our setup $LL\bar{e}$ obtains a vanishing vev right after inflation so that the amplitude of the magnetic field is frozen during the radiation epoch, evolving as $B \sim a^{-2}$. Since the temperature scales as the inverse of the of the scale factor, the magnetic field at the time of galaxy formation is given by $B_{l,0} = B_l(T_{gf}/T_R)^2$, where T_{gf} is the temperature at the galaxy formation and T_R is the reheat temperature. The length scale also evolves, $l \sim a$, so the length scale at the galaxy formation is related to the one at the end of inflation by $l_{gf} = l(T_R/T_{gf})$. Putting everything together we obtain the amplitude of the magnetic field at the time of galaxy formation as

$$\begin{aligned} B_{l,gf} &\approx \cos\theta_W C(\nu')\left(\frac{1}{2}-\nu'\right) H_I^{1/2+\nu'}\ l_{gf}^{\nu'-3/2} \\ &\quad \times \left(\frac{T_{gf}}{T_R}\right)^{1/2+\nu'} . \end{aligned} \quad (17)$$

We estimate the reheat temperature by assuming that the inflaton potential energy density is directly converted to a thermal bath, $V_{end} \approx 3M_p^2 H_I^2 \approx (\pi^2/30)g_* T_R^4$, where $g_* = 106.75$, is the number of relativistic degrees of freedom. Thus we obtain an order of magnitude estimate for the magnetic field which reads

$$\begin{aligned} B_{l,gf} &\approx 7\cdot 10^{-30}{\rm G}\ (1+{\rm z_{gf}})^{-1}\left(\frac{\rm g'}{0.01}\right)^2\left(\frac{|\phi|}{\rm H_I}\right)^2 \\ &\quad \times \left(\frac{T_0}{2.73\ {\rm K}}\right)\left(\frac{1\ {\rm kpc}}{l_{gf}}\right)\left(\frac{V_{end}^{1/4}}{10^{16}\ {\rm GeV}}\right) , \end{aligned} \quad (18)$$

when $(1/2-\nu') \approx (g'|\phi|/H_I)^2 \ll 1/2$ and $T_{gf} = (1+z_{gf})T_0$, where z_{gf} is the redshift of the galaxy formation time and a factor of $5\cdot 10^4(1+z_{gf})^{-2}$ [18] due to the increase of flux at the collapse of protogalaxy is included.

Note that the the strength of the magnetic field depends on the vev of the hypermangetic field during inflation. Obviously when the vev is zero, the conformal invariance is restored and there is no magnetic field. For a field almost massless during inflation, we would naturally expect that the MSSM flat direction vev is locked to the expansion of the Universe such that $|\phi| \sim H_I$. This yields the maximum amplitude of the primordial magnetic field to be of order $10^{-30}\ G$ with a coherence length $l \sim 1$ kpc as required in [19].

A word of caution, though: the above estimation of the strength does not take into account of the sub-horizon plasma physics due to charge separation and inhomogeneous current distribution, see for instance [20]. Note that MSSM flat directions do not generate currents at any time but nevertheless the current density obtains non-vanishing fluctuations, which requires a detailed analysis of the sub-horizon effects which goes beyond the scope of the present paper. On the other hand the condensate is assumed to decay instantly at the end of inflation, so that conformal invariance of the gauge

field is restored. In reality one would expect the condensate to be stuck at its inflationary value until the Hubble parameter is of the order of its mass ~ 1TeV. Thus the magnetic field amplitude decay rate is slower than in the conformally invariant case. Also in reality the condensate should be slow-rolling during and after inflation unless it is at the minimum of the potential, but we expect that this produces only minor quantitative changes to our results.

In conclusion, we have provided a natural mechanism within supersymmetry for generating seed magnetic field based on generating non-vanishing masses to the SM gauge fields from the vev of the MSSM flat directions. The simplicity of our scenario lies in that we do not break conformal invariance by an ad-hoc assumption, nor do we invoke any exotic fields or coupling other than the ones present in the minimal extension of the SM. The excited gauge fields, mainly the hypercharge field arising from $LL\bar{e}$ flat direction, are stretched during inflation outside the horizon in a similar way as the scalar fluctuations responsible for generating the perturbations in the cosmic microwave background radiation. The subtle point here is that we generate non-vanishing current density required for breaking the conformal invariance through the fluctuating current.

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